# *Electrostatic lens sizing*



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Revision C1: complete rewriting.

## Abstract

The goal of this presentation is to give some information about the sizing of electrostatic lenses, mainly the focal length. These lenses are used to focus particles beams. It Is proposed a small program and formulas taking account different parameters (voltages and configuration) in a relative simple way.
This presentation relies on a personal simulator.

A physical explanation of the focusing principle is proposed. It is afterwards explained why a negative potential will also focus a ions beam, even it seems counter-intuitive. Moreover, it is described the expected behavior of lenses in presence of a strong space charge, or in presence of two different types of plasma (hot ions/cold electrons plasma and fusion neutral plasma).


## 1. Goal, presentation and notations used

The goal of this presentation is to give some additional information about the sizing of electrostatic lenses (i.e. focal length), used to focus particles beams.

The main need of the author was to rapidly size simple lenses, but the different documents found from Internet did not enter in details and did not permit to size the lenses according to the different parameters (voltages and configuration) in a relative simple way, without simulations. The second need is to understand how lenses physically work.

This presentation relies on the Multiplasma simulator program version 1.12 (not public) developed by the author and used for the simulation of the lenses (among other functions).

So this paper will permit to size an electrostatic lens by giving an order of magnitude of the focal length.

### Notations

- The simple product is indicated with « x » or « . ».
- “<<” for “very inferior” and “>>” for “very superior”.
- |x|, absolute value of x.
- The sign “~” is worth for “proportional”.
- A vector (as the electric field **E**) is in bold but its components in a cylindrical coordinate system (Er, Eφ, Ez for example) are scalar so in light. Note that, due to the azimuthal symmetry, the azimuth coordinate (Eφ) does not need to be considered.

The author uses SI units or sub-multiple: the unit of length taken here is the “mm” which is more convenient that the “m” for lenses.

<u>Note</u>: breakdown problem between electrodes is not addressed.

Hypothesis:

- The relativity is not considered, the particle speed being supposed far inferior to the speed of light.

- The lens is supposed installed in the middle on a straight pipe which length is at least twice the focal length.

- It is supposed that the initial particle trajectory is horizontal. However due to the not nil beam emittance, the beam is normally more or less divergent, so with a non nil radial speed. However, this one is not considered in this paper.

- About space charge
  It is supposed that the space charge can be neglected. Note that if the space charge is large, its effect is predominant and the lens is without any use.
  The maximum “divergent” space charge radial electric field Ec (V/m) can be determined for a current I (A) of particles beam of radius R (m) and speed v (m/s), by the formula: $Ec = \frac{I}{2 \times \pi \times \epsilon 0 \times \mathrm{v} \times \mathrm{R}}$ with ϵ0=8.85E-12 F/m
  This value of Ec must be very inferior to the “convergent” radial electric field Er generated by the lens. A very rough estimation would be based on a U0 equal to Ul/2 (see figure 2). So the rough mean radial Er along the radius would be equal to Ul/(2*Rint).
  If Ec<<Er the lens will make converge the beam towards the axis and the proposed formulas will be of some use. Otherwise the trajectory of the beam will be influenced by the space charge and the proposed formulas are not applicable. The reference [1] can be of interest to take into account space charge.

  In figure 1, two typical trajectories are given:

  - The standard one in black, without space charge. Note that this trajectory is conventional, because the real trajectory looks like the one shown in figure 5.

  - The trajectory in red, with space charge. The ion cannot reach the axis because the space charge electric field opposes a force to the particle.

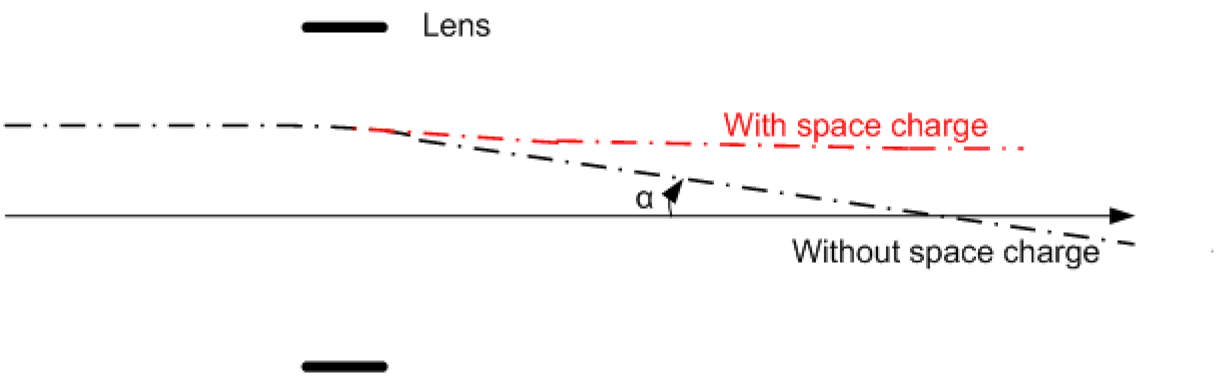



**Figure 1: typical trajectory with and without space charge**

## 2. Description of the simple lens

The goal of the lens is to focus a beam. Due to the interest of the author, it will be supposed that the particles are ions with one charge (D+ or T+). But it could be electrons with opposed voltages. Below the figure 2 shows the different notations. The simple lens is in fact a single electrode,  in form of washer. This type of lens can also be called “Wehnelt”.

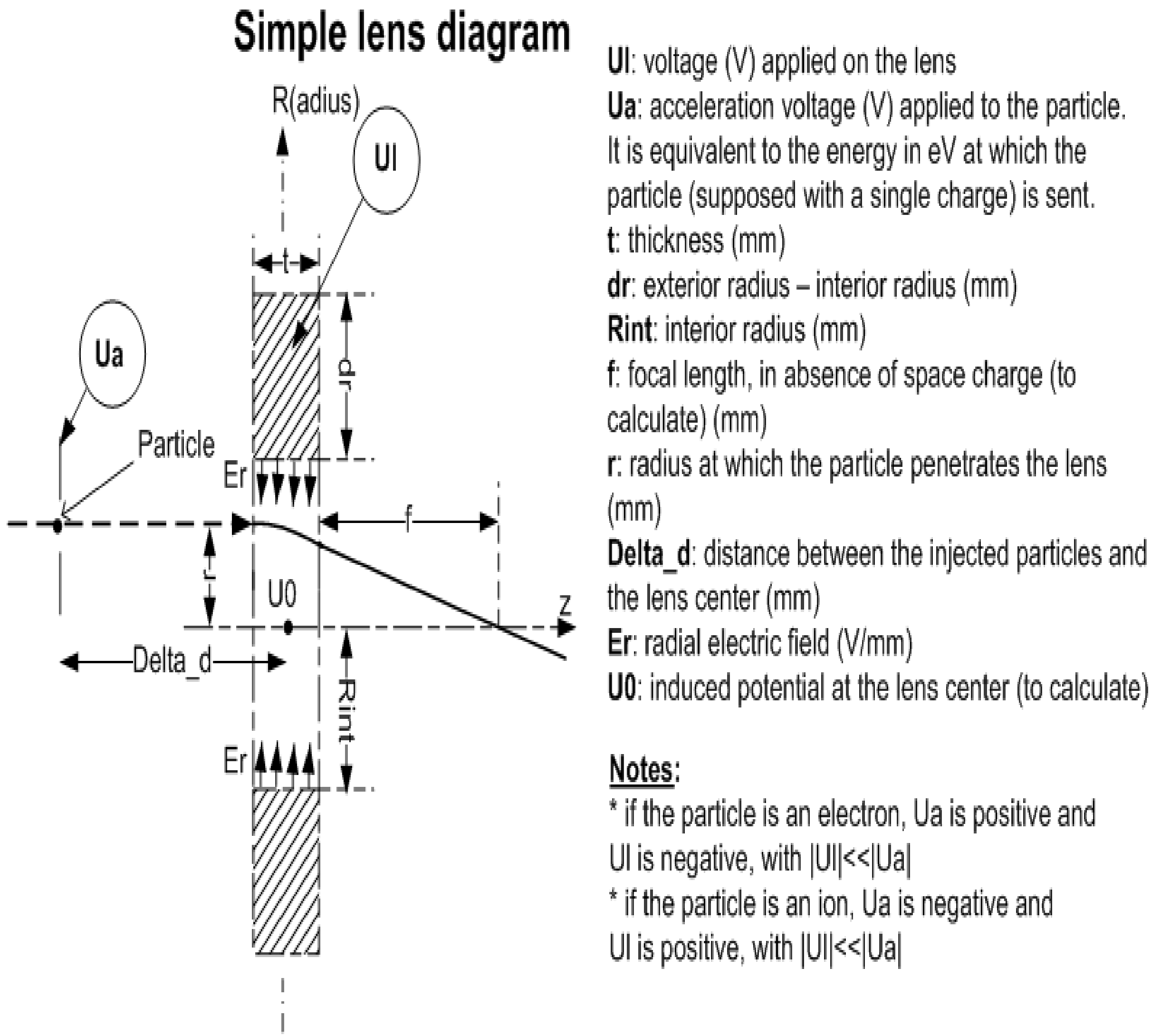



**Figure 2: simple lens diagram**

## 3. Simple lens sizing

It will be, first, determined a simplified theoretical formula for the focal length, with the first following hypothesis:

- As yet said, the lens has a cylindrical symmetry, so the azimuthal behavior is not taken into account.
- The initial trajectory of the particle is horizontal (along and above the z axis),
- The distance “r” to the axis is very small compared to the lens radius (Gaussian hypothesis). r is positive by agreement, the axis R carrying “r” being upwards. With this Gaussian hypothesis, the focal length will not depend on “r”. In the reality r must be, roughly, inferior or equal to the half of the lens radius. Beyond, the beam converges with different focus lengths (causing aberrations).

It can be demonstrated that the trajectory of the ion obeys, at a given point i, to:

$$\left(\frac{d^2r}{d^2z}\right)i = \frac{-q}{m \times vzi^2} \times \left[\frac{\delta V}{\delta r} - \frac{\delta V}{\delta z} \times \left(\frac{dr}{dz}\right)i\right]$$

With r the radius, z the axial distance along the z axis, q the charge in Coulomb, m the mass in kg, vzi, the axial speed vz at the point i, V the potential (which depends on r, z and Ul). For details, look at the reference [1], page 15.

To simplify, it will be done the hypothesis that dr/dz (i.e. the angle of the trajectory) will be always very small so as to neglect the second term. This supposes that Ul/Ua will be small.

So the expression can be simplified in: $\left(\frac{d^2r}{d^2z}\right)i = \frac{-q}{m \times vzi^2} \times \frac{\delta V}{\delta r}$

Note that: $\frac{\delta V}{\delta r} = -Er$ (Er the radial electric field, depending on r, z and Ul)

In a general way, it is reminded that the electric field **E** (vector) is the reverse of the potential (scalar) gradient, i.e. **E** =-grad(V).

In addition, it can be demonstrated via the divergence theorem (i.e. div(**E**)=-ΔV=0, “=0” because the space charge is negligible, as supposed) that:

$$\frac{\delta V}{\delta r} = 0.5 \times \frac{\delta Ez}{\delta z} \times r$$

With Ez the electric field along z (Ez depending on r, z and Ul). For details, look at the reference [1], page 17.

Moreover, $m \times vzi^2 = 2 \times Eki(J)$ (Eki being the kinetic energy in J at the point i).

Eki can be expressed in eV rather than in Joule, with $Eki(eV) = \frac{Eki(J)}{q}$

with the charge q=1.602E-19 C (for an ion D+ or T+).

It follows: $\frac{m \times vzi^2}{q} = 2 \times Eki(eV)$ So $\frac{q}{m \times vzi^2} = \frac{1}{2 \times Eki(eV)}$

Now, look at the induced potential in V on the Figure 3. It appears clearly that, circulating from left to right along z (horizontal axis), the potential induced by the lens

increases from 0 to U0 at the center of the lens and then decreases down to 0. Consequently, the electric longitudinal field Ez is always negative before the lens (Ez called Ez1), and positive after the lens (Ez called Ez2), with Ez2=-Ez1, Ez changing of sign at the center of lens: ←---Ez1---- Lens (center) ----Ez2----→
Note that Ez1 slows down ions and Ez2 speeds up ions.

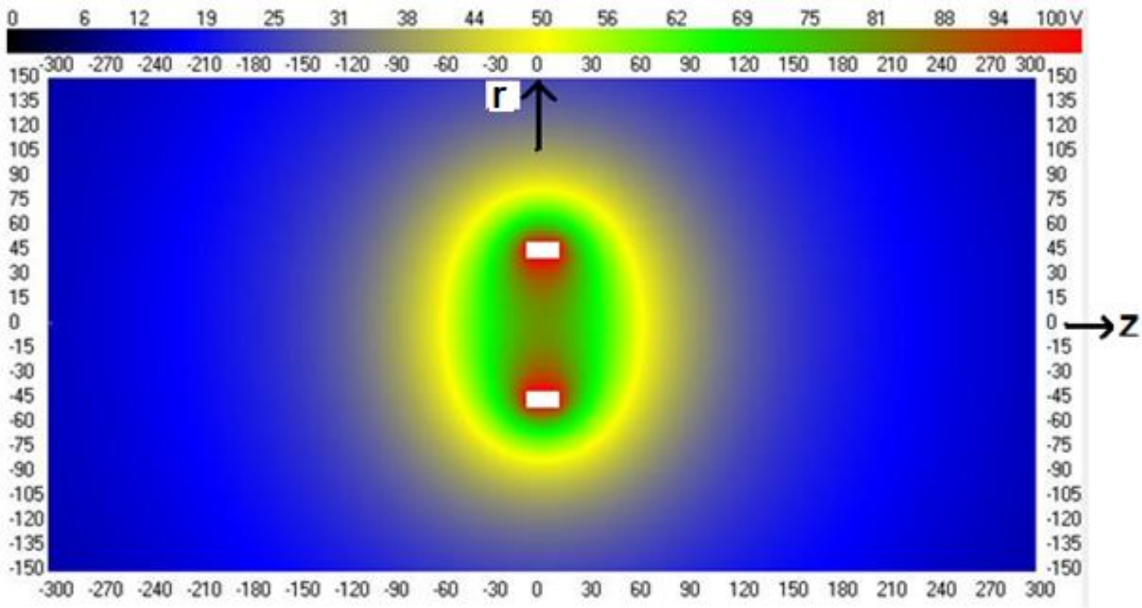



**Figure 3: snapshot of an equipotentials display of a simple lens**

Below on the figure 4 are given typical evolutions of the induced voltage U, the radial and axial electric fields (Er and Ez) for a simple lens, obtained for a trajectory along and above the z axis. The radial and axial electric field sharing the same unity, note that Ez>>Er. Note also that Ez=0 for z=0, which is logical due to the lens symmetry. The positive maximum of Ez (Ezmax) is obtained at z1. By symmetry the negative maximum (-Ezmax) is obtained at –z1. Ez1 represents Ez before z=0 and Ez2 represents Ez after z=0.

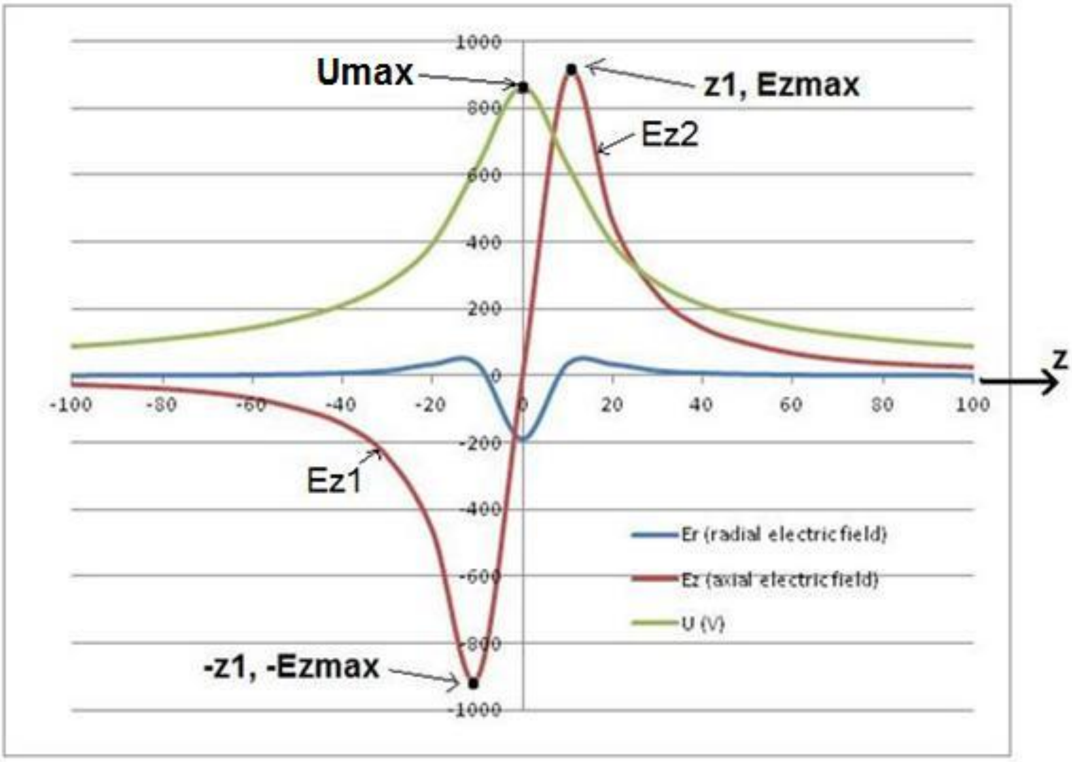



**Figure 4: typical evolutions of U, Er and Ez for a trajectory along and above the z axis**

So an ion accelerated by an electrode at Ua will be decelerated by Ez1 until reaching z=0. Afterwards, it will be accelerated by Ez2. If the induced voltage at the point i is equal to Ui (Ui depending on r, z and Ul), the ion energy at point i will be equal to $Eki(J) = q \times (Ua - Ui)(V)$ or $Eki(eV) = (Ua - Ui)(V)$

So with all these considerations, it can be written:

$$\left(\frac{d^2r}{d^2z}\right)i = \frac{-0.5 \times r}{2 \times (Ua - Ui)} \times \frac{\delta Ez}{\delta z} = \frac{-r}{4 \times (Ua - Ui)} \times \frac{\delta Ez}{\delta z}$$

As $\frac{dr}{dz}$ is obviously nil when z →-∞ knowing the exact value of U and Ez in any point, a precise calculation of the trajectory at any point would consist to integrate $\frac{d^2r}{d^2z}$ twice between -∞ and the point i, to get $\left(\frac{dr}{dz}\right) i$ and r, and finally the focal length f and all the parameters of the trajectory. Of course, such calculations might be done numerically on a computer.

As an example, below are given, on the figure 5, typical evolutions of the radius r from the initial position of the ion, its radial speed (vr) and its axial speed (vz).

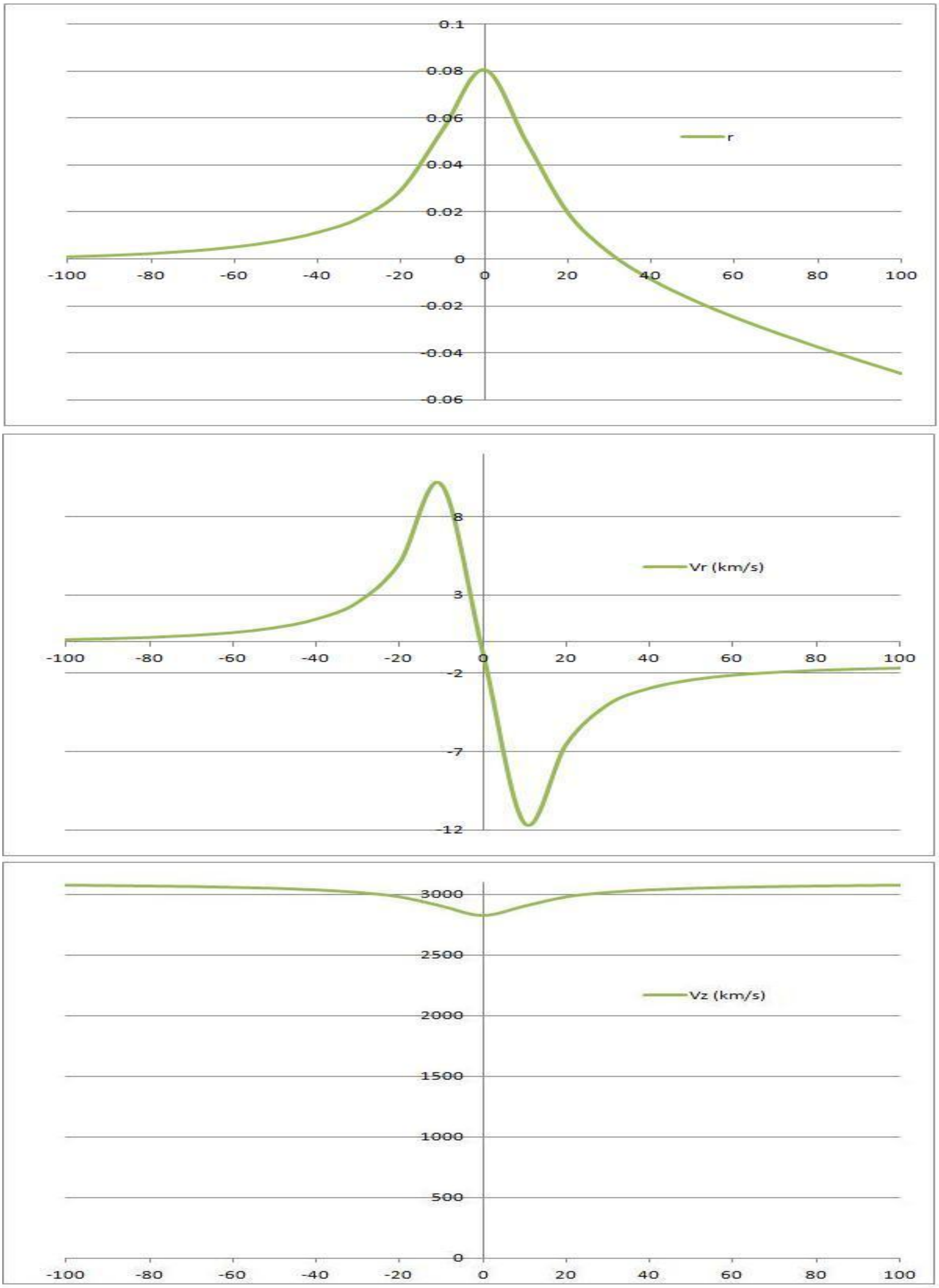



**Figure 5: typical ion trajectory defined by its radius position r (top), its radial speed vr and its axial speed vz (bottom) for Ul>0**

An approximate solution for the focal length f is given in reference [2] page 116:

$$f = \frac{4 \times Ua^2}{Ez \times Ul}$$

Here “Ez” is a characteristic value of the axial electric field. It must be determined for such simple lens.

Now, as Ez is proportional to Ul, it follows that f is proportional to $\left(\frac{Ua}{Ul}\right)^2$.

Ez is not known but it can be determined experimentally. Moreover for a given lens, U0=K.Ul (K determined experimentally according to t, Rint and dr).

About Ez, experimentally, at $D1 = \left[2 \times \left(Rint + (\frac{2}{3} \times dr)\right)\right] + \left(\frac{1}{2} \times t\right)$ from the lens center, the induced voltage is relatively constant around 0.416 x U0 for the considered lenses. This gives a reference $Ez0 = \frac{0.584 \times U0}{D1}$.

For an Ez different from Ez0, it depends mostly on the distance Delta_d. It is equal to $Ez = K2 \times Ez0$, It remains to determine the constant K2 experimentally (according to Delta_d, Rint and dr).

Finally, $f(mm) = \frac{4 \times Ua^2 \times D1 \times K^2}{K2 \times 0.584 \times U0^2}$

In the program the constant $\frac{4 \times K^2}{K2 \times 0.584}$ is replaced by the constant 3.36*K1, so $f(mm) = \frac{3.36 \times K1 \times Ua^2 \times D1}{U0^2}$, but it comes to the same. K1 is determined to match, at best, the simulations (supposed to exactly correspond to the reality) with the calculated f.

The complete calculation with the experimental formulas for U0 and K1 is given in the Pascal (Delphi 6) procedure in Appendix.

So given Ul, Ua, Rint, dr, t and Delta_d, it will be given the probable focal length (f) with an estimated dispersion (compared to simulations) from 1/2 to 2 times the result.

An example is given in the program in Appendix.

The limits of validation are the following:

- Delta_d between 1 and 10 times the lens exterior diameter. Note that the influence of the acceleration electrode at the voltage Ua is not taken into account (neglected),
- dr inferior or equal to Rint
- t inferior or equal to 2*Rint
- Ul<=0.4*Ua

**Physical explanation of the proportionality of f with $(Ua/Ul)^2$**

It is reminded that only ions are implicitly considered.
Intuitively, it seemed natural, for the author, to think that the focal length f be proportional to Ua/Ul for the following (bad) reason:

Er (the radial electric field at lens level, convergent so negative) is proportional to Ul-U0 and finally to Ul (U0 being proportional to Ul).
So the radial speed $Vr = K \times Er \times cd$ with "cd" for "crossing duration".
$\mathrm{cd} = \frac{t}{vz}$ (t: thickness and vz: axial speed along z). Hence $vr \sim \frac{Ul}{vz}$
The angle $\alpha = \frac{vr}{vz} \sim \frac{Ul}{vz^2} \sim \frac{Ul}{Ua}$ (because $vz \sim \sqrt{Ua}$)
And finally $\mathrm{f} = \left|\frac{r}{\alpha}\right| \sim \frac{Ua}{Ul}$
But it is not true, because f is really proportional to $\left(\frac{Ua}{Ul}\right)^2$

Looking at the evolution of the radial electric field Er along the trajectory in figure 3, it can be noted that for the most part of the trajectory the electric field is low and positive (divergent), whereas at the lens level, the radial electric field (Er) is high and negative (convergent).
On the figure 6 of the next page, it is displayed the simplified shapes of the electric field Er, its integration along the trajectory U=ΣEr . dz and the radial speed vr (simplified, compared to vr in figure 5).

Hypothesis

- The ion is supposed to initially move axially on a radius r0. The variation of r when the ion crosses the lens is very small. So r(z=0) ≈ r0.

- In a general way, if the induced voltage at the point i is equal to Ui, the ion energy at point i will be equal to $Eki(J) = q \times (Ua - Ui)(V) = \frac{m \times Vi^2}{2}$
With vi the ion speed. As the radial speed vri is very small compared to vi, the axial speed vzi can be assimilated to vi. So: $vzi = \sqrt{\frac{2 \times q}{m}} \times \sqrt{(Ua - Ui)}$
- In a general way from the elementary formulas, $vr = \gamma \times t$ and $Fr = q \times Er = m \times \gamma$ with q the ion charge, m the ion mass, Fr the radial force, γ the radial acceleration and t the elapsed time, it can be deduced that: $\gamma = \frac{q}{m} \times \mathrm{Er}$ and $\mathrm{vr} = \frac{q}{m} \times \mathrm{Er} \times \mathrm{t}$

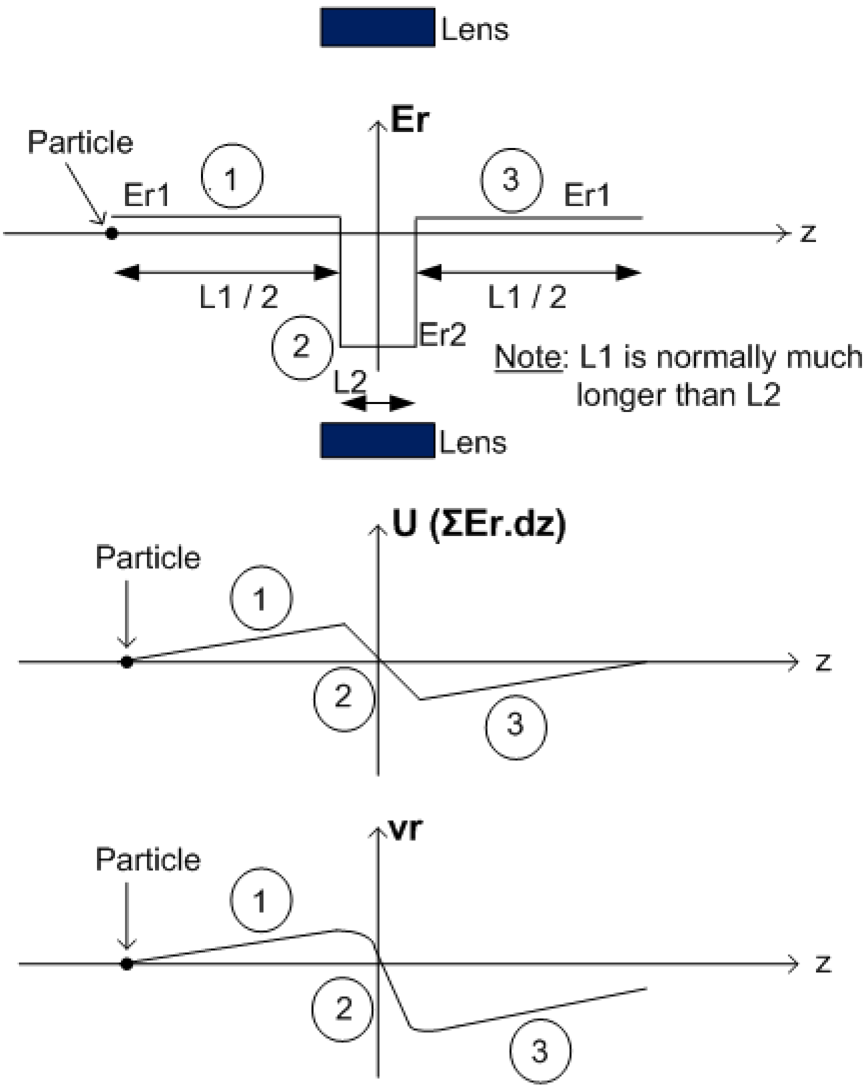



**Figure 6: simplified radial electric Er (top), integration of Er along z (called U) and radial speed Vr (bottom)**

It can be noted that:

$$\sum Er1 \times dz = -\sum Er2 \times dz \; so \; Er1 \times L1 = -Er2 \times L2 \; or \; Er2 = \frac{-Er1 \times L1}{L2}$$

The parts 1 and 3 (Er1>0) are crossed by the particle at the mean speed vz1 along z:

$vz1 = \sqrt{\frac{2\times q}{m}} \times \sqrt{(Ua - Umean)}$ Umean is a priori unknown but it is of course inferior to Uz0, Uz0 being the induced voltage at z=0, r≈r0.

So the time t1 to cross the parts 1 and 3 is equal to $t1 \sim \frac{L1}{vz1}$.

The total positive speed vr1 (upwards) got by the particle when it crosses the parts 1 and 3 is equal to $vr1 = \frac{q}{m} \times Er1 \times t1 = \frac{q}{m} \times Er1 \times \frac{L1}{vz1}$

The part 2 (Er<0) is crossed by the particle at the approximate mean speed vz2 along z: $vz2 = \sqrt{\frac{2\times q}{m}} \times \sqrt{(Ua - Uz0)}$ with U0<Uz0<Ul.

So the time t2 to cross the part 2 is equal to $t2 \sim \frac{L2}{vz2}$.

The total negative speed vr2 (downwards) got by the particle when it crosses the part 2 is equal to $vr2 = \frac{q}{m} \times Er2 \times t2 = \frac{q}{m} \times Er2 \times \frac{L2}{vz2}$

So $\mathrm{vr2} = -\frac{q}{m} \times \left(\frac{\mathrm{Er1}\times\mathrm{L1}}{\mathrm{L2}}\right) \times \frac{\mathrm{L2}}{\mathrm{vz2}} = -\frac{q}{m} \times \mathrm{Er1} \times \frac{\mathrm{L1}}{\mathrm{vz2}}$

The final radial speed (after the part 3) vr_final is such that:

$$vr\_final = vr1 + vr2 = \frac{q}{m} \times Er1 \times L1 \times \left(\frac{1}{vz1} - \frac{1}{vz2}\right)$$
$$= \frac{q}{m} \times Er1 \times L1 \times \left(\frac{vz2 - vz1}{vz2 \times vz1}\right)$$

Noting that Uz0 and Umean <<Ua and so using $\sqrt{(1 - Epsilon)} = 1 - \frac{Epsilon}{2}$, it comes after several calculations, neglecting Umean and Uz0 in front of Ua:

$vr\_final = \sqrt{\frac{q}{m}} \times \frac{Er1\times L1}{2\times\sqrt{2}} \times \frac{(Umean-Uz0)}{Ua^{1.5}}$

Er1>0 and (Umean-Uz0)<0 so vr_final is negative and the beam is convergent (towards the axis).

The angle α of the particle trajectory is equal to:

$\alpha = \frac{vr\_final}{vz\_final}$ with $vz\_final = \sqrt{\frac{2\times q}{m}} \times \sqrt{Ua}$

So $\alpha = Er1 \times L1 \times \frac{(Umean-Uz0)}{4\times Ua^2} = -Er2 \times L2 \times \frac{(Umean-Uz0)}{4\times Ua^2}$ (formula 1)

$Er1 \sim U0 \ or \ Ul.$

Now $\mathrm{Umean}$ and $\mathrm{Uz0} \sim U0 \ or \ Ul$, but (Umean-Uz0)<0 so:

$$(\mathrm{Umean} - \mathrm{Uz0}) \sim -U0 \ or - \ Ul$$

Finally $\alpha \sim -\left(\frac{U0}{Ua}\right)^2 \sim -\left(\frac{Ul}{Ua}\right)^2$

So f (focal length)=|r/ α| is proportional to $\left(\frac{Ua}{Ul}\right)^2 \ or \left(\frac{Ua}{U0}\right)^2$, as expected.

<u>Focusing principle</u>

From the previous considerations, it appears that the focusing is due to the variation of axial speed vz along z (see figure 6). Thanks to the strong deceleration at lens level (part 2 of figure 6), the convergent radial speed is applied for an increased time whereas the divergent radial speed (on parts 1 and 3 of figure 6) is also applied for an increased time by the deceleration but not as strongly as at lens level. Consequently, the amplitude of the convergent radial speed (vr<0) due to Er>0 at lens level is superior to the amplitude of the divergent radial speed (vr>0) due to Er<0 outside the lens, so the global effect is a convergent radial speed (vr<0) .
This is illustrated with the vr evolution on figure 6.

Effect of a lens carried to a negative voltage Ul

Let's study the effect of a negative voltage Ul instead a positive one.
From the previous result above, it can also be deduced that :

$$\alpha = Er1 \times L1 \times \frac{(Umean - Uz0)}{4 \times Ua^2}$$

but in this case Er1<0 instead Er1>0 and (Umean-Uzo)>0 instead (Umean-Uzo)<0 .
But α will stay negative and so the beam will stay convergent. Even if it seems counter-intuitive, a lens carried to a negative potential will make converge a beam of ions (confirmed by [2]).

As an example, below are given, on the figure 7, typical evolutions of the radius r from the initial position of the ion, its radial speed (vr) and its axial speed (vz) for Ul<0.
As this Ul has a value just reverse from the one used in figure 5 (-10 kV versus 10 kV), these 2 figures can be compared.

Now several simulations done by the author show that a potential Ul<0 on a lens seems less efficient than a potential Ul>0.

Consequently, for any non nil voltage (negative or positive), and for any particle with charge (electrons or ions), no matter the axial direction (from left to right or reversely), the lens will make converge the beam.
For a beam accelerated up to Ua, this convergence will increase with the voltage Ul applied on the lens, according to the law $\alpha \sim -\left(\frac{Ul}{Ua}\right)^2$

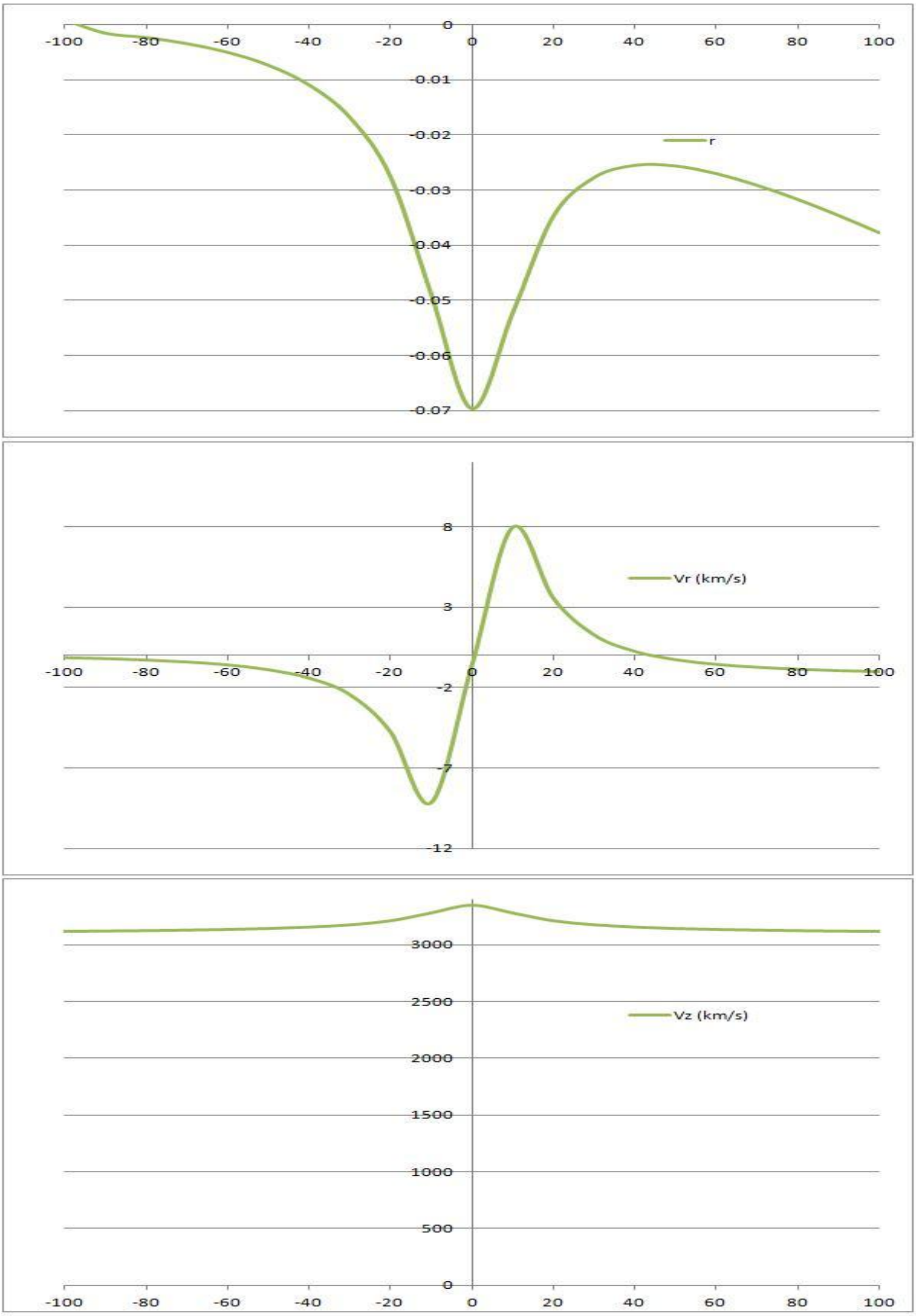



**Figure 7: typical ion trajectory defined by its radius position r (top), its radial speed vr and its axial speed vz (bottom) for Ul<0**

## 4. About lenses with 0V symmetrical electrodes

It can be simpler to force the electric field with two electrodes at 0 V, symmetrically disposed. The number of parameters being high, it will not be proposed a sizing, but two examples (type 1 and type 2).

The big advantage of that type of lens (called “Einzel” lens) is that it does not depend much on the distance between the injected particles position and the lens center (Delta_d), contrary to the single lens. It is almost an isolated system.

On figure 9, it will be found the diagram of these two examples of lenses.

Below, on figure 8, is an example of equipotentials for such lenses (in term of induced potential in V). It is obvious, compared to the previous one for a simple lens (figure 3) that the electric field is clearly limited by the two symmetrical electrodes at 0 V.

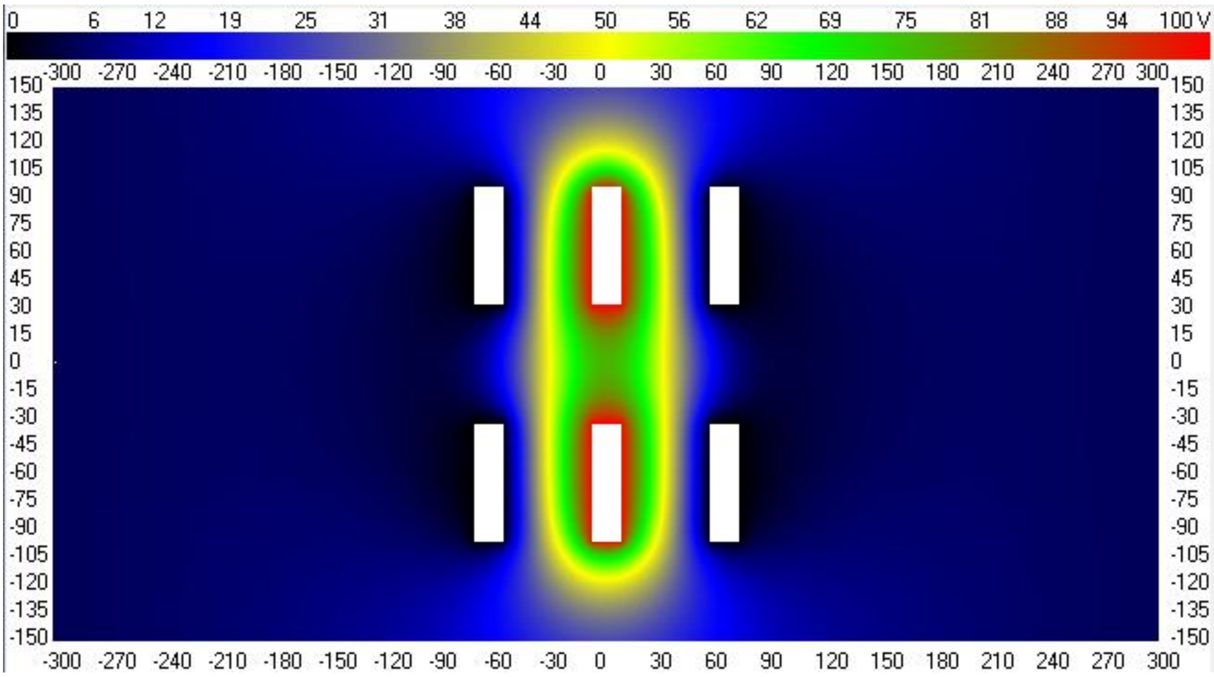
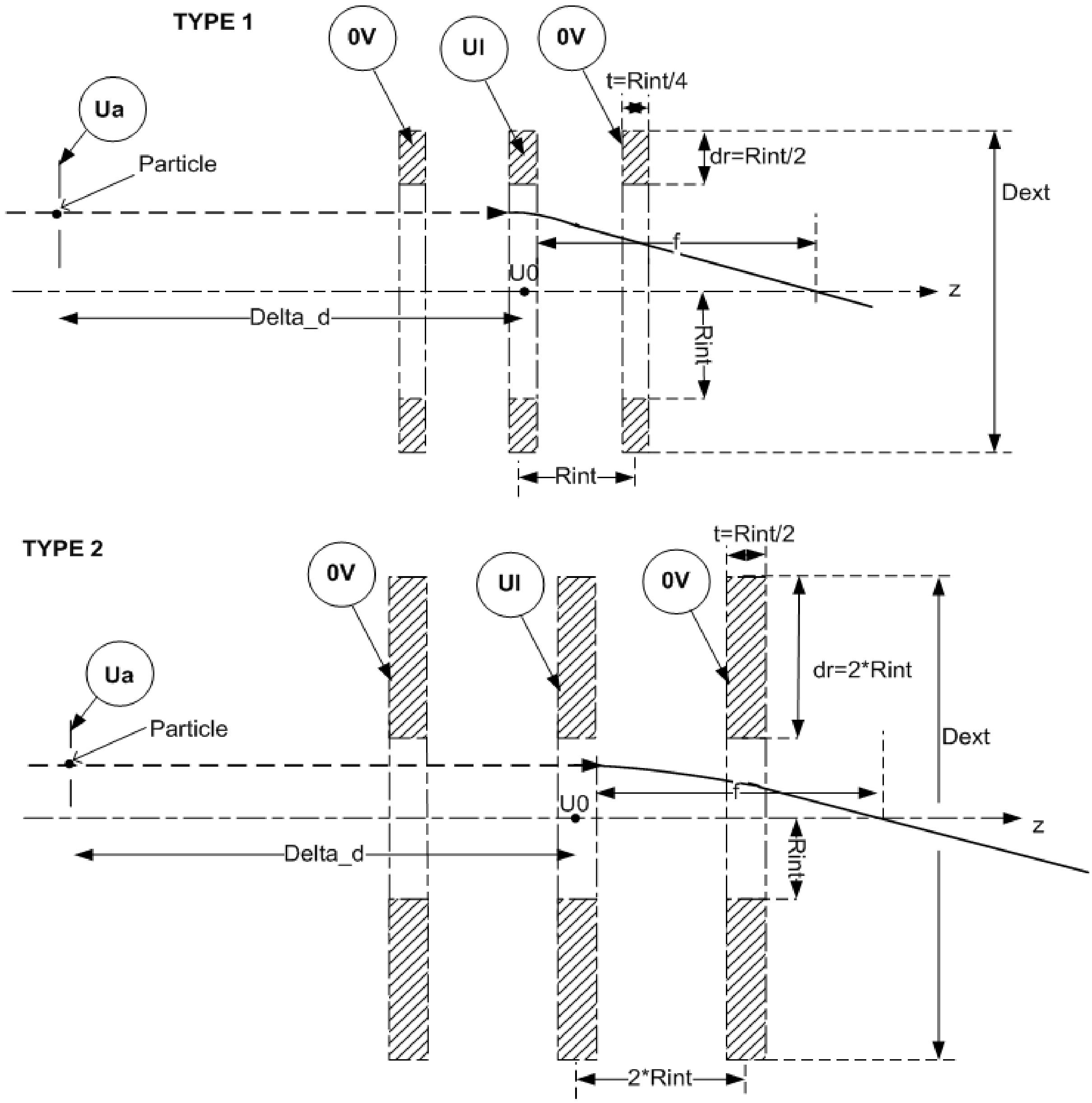



**Figure 8: snapshot of an equipotentials display of a lens with 0V symmetrical electrodes**

## Two examples of lenses with symmetrical 0 V electrodes




**Ul**: voltage (V) applied on the lens
**Ua**: acceleration voltage (V) applied to the particle. It is equivalent to the energy in eV at which the particle (supposed with a single charge) is sent.
**t**: thickness (mm)
**dr**: exterior radius – interior radius (mm)
**Rint**: interior radius (mm)
**Dext**: lens external diameter (mm)
**f**: focal length, in absence of space charge (to calculate) (mm)
**Delta_d**: distance between the injected particles and the lens center (mm)
**U0**: induced potential at the lens center (V)

**Notes:**
* if the particle is an electron, Ua is positive and Ul is negative, with |Ul|<<|Ua|
* if the particle is an ion, Ua is negative and Ul is positive, with |Ul|<<|Ua|

**Figure 9: lenses with 0V symmetrical electrodes**

The focal length f1 for the type 1 is given by this experimental formula:

$$\mathrm{f1\ (mm)} = 147 \times \left(\frac{Rint}{8}\right) \times \left(\left(\frac{Delta\_d}{3 \times Dext}\right)^{0.1}\right) \times \left(\frac{Ua}{Ul}\right)^{2}$$

As it can be seen, due to the small 0.1 power factor, the influence of Delta_d is weak (but still monotone).

The focal length f2 for the type 2 is given by this experimental formula:

$$\mathrm{f2\ (mm)} = 40 \times \left(\frac{Rint}{4}\right) \times \left(\frac{Ua}{Ul}\right)^{2}$$

As it can be seen, the influence of Delta_d is not taken into account because it is weak (< to 15%) and, overall, not monotone.

## 5. Lenses used for different types of plasma

### 5.1 Plasma composed of “hot” ions and “cold” electrons

The plasma is supposed composed of a beam of ions at high energy moving axially in a “sea” of “cold” electrons (i.e. of very small energy), in an almost neutral plasma. A way to focus the ions beam is to use a magnetic field to insulate electrons. In that case, the lens is called an “electrostatic plasma lens”. The magnetic lines of force and the electrostatic equipotentials must have the same shape. It is not used a simple lens but an “Einzel” lens with much more than 3 electrodes.

<u>Effect of an “electrostatic plasma lens”</u>
For the effect of such lens, see [4] for a review article about this equipment, which shows that the compression factor can be very interesting.

### 5.2 Fusion neutral plasma

A fusion plasma is composed of electrons, D+ and T+ ions. It is neutral as there are as many electrons as ions, the global charge being nil. Thanks to collisions between particles, electrons and ions are in thermal equilibrium at about the same mean energy E (with in general about E=15 Kev per particle).

Particles are thermalized, i.e. their speeds follow a Maxwell Boltzmann isotropic distribution. 2/3 of their energy is radial and 1/3 is axial. So the mean arithmetic speed of both ions and electrons beams is nil whereas the mean quadratic speed in each direction is not nil. It means that particles will move on the circuit in one direction or the other, randomly.

The plasma is confined in the main circuit thanks to a powerful magnetic axial field B, with B about 4 T in a modern reactor (Tokamak or Stellarator).
Note that lenses are not used on the main circuit of these fusion reactors.

Effect of a lens on a fusion neutral plasma

Due to the very weak electrostatic pressure compared to the magnetic pressure, the sole lens is, in any case, unable to confine a fusion plasma. So the lens is supposed used, with the normal axial magnetic field.

A simulation has been done on such plasma with a simple lens carried on positive and negative voltages, the magnetic field being axial..

According to this simulation, it appears that there is not the same focusing effect as the one described in §3. Indeed, it is not possible to focus electrons and ions in the same time, but only one species (i.e. electrons or ions). For example, a positive voltage will focus ions and un-focus electrons, and reversely.

This can be explained in the following way.
One hypothesis taken for §2 to §4 is that the initial direction of the particle is axial. Now, in case of a thermalized particle, the mean speed on each direction is nil. This corresponds to a Brownian motion where the particles move randomly around a mean position. The particles, as almost immobile charges, are simply attracted by the opposite polarity of the lens or repelled by the same polarity. For example, a positive voltage will repel (so focus) ions and attract (so un-focus) electrons, and reversely.

Very close to the lens wall, it is formed a “Debye sheath layer” (see [3] or Wikipedia). So a positive potential will create an electrons sheath layer with more electrons lost than ions lost. Reversely, a negative potential will create an ions sheath layer with more ions lost than electrons lost.

To conclude, this effect could be used as a possible control of the particles lost and therefore as a control of the electrons/ions densities, but not for a focusing of all particles.

## 6. Conclusion

In §2 and §3, a simple lens has been studied. The focal length estimate (f) of the single lens, in absence of space charge, is proposed in Appendix

Moreover in §3 is given a physical interpretation of the focusing.

The focal lengths of the type 1 and type 2 lenses with 0 V symmetrical electrodes are proposed in §4.

The dispersion of the result (compared to a simulation which has also a certain margin of error) is estimated to be between 1/2 and 2 times the result.

Moreover, in §1 it is given the type of effect on the particle trajectory due to a strong space charge, whereas in §5 is analyzed the behavior of lenses for two different types of plasma.

## 7. References

[1] « Etude théorique et expérimentale de la focalisation des ions afin d'améliorer la brillance du faisceau ionique par suppression des causes d'aberrations » by Jean Faure

[2] « Sur une nouvelle méthode de focalisation des faisceaux d'ions rapides. Application à la spectrographie de masse » by Louis Cartan

[3] Book « Plasmas collisionnels » by Michel Moisan and Jacques Pelletier, edited by EDP sciences, 2014

[4] "Invited Review Article: The electrostatic plasma lens"
Rev. Sci. Instrum. 84, 021101 (2013); https://doi.org/10.1063/1.4789314

# APPENDIX

```
PROCEDURE SIMPLE_LENS_SIZING_FOR_IONS;
VAR t:SINGLE;{"t" for "thickness of the lens" in mm}
VAR Ul:SINGLE;{voltage (in V) applied on the lens (positive for ions and negative for electrons)}
VAR Ua:SINGLE;{accelerator voltage (in V) applied on the particles (negative for ions and positive for electrons)}
VAR Rint:SINGLE;{interior radius in mm}
VAR Dext:SINGLE;{exterior diameter in mm}
VAR dR:SINGLE;{exterior radius - interior radius in mm}
VAR U0:SINGLE; {induced potential at the lens center (to calculate)}
VAR Delta_U:SINGLE;{|U1-U0| in V}
VAR D1:SINGLE;{standard distance compared to the lens center in mm}
VAR Delta_d:SINGLE;{distance between the injected particles and the lens center in mm}
VAR K1:SINGLE;{coefficient to take into account Delta_d, for the Ez estimation}
VAR f:SINGLE;{focal length to calculate in mm}
BEGIN
 {set of hypothesis, for ions}
 Delta_d:=99;
 Ul:=40000;
 Ua:=-100000;
 dR:=2;
 Rint:=9;
 t:=4;

 {not permitted}
 IF (Ul<0) OR (Ua>0) THEN EXIT;
 IF ABS(Ul)>ABS(Ua) THEN EXIT;

 {calculation}
 Delta_U:=Ul*0.2479*Power(2.25*t/Rint,-0.2863-(t/Rint)*0.4657)*Power(4.5*dR/Rint,-0.2852-(dR/Rint)*0.0891);
 U0:=Ul-Delta_U;
 Dext:=2*(Rint+dR);
 D1:=2*(Rint+2/3*dr)+t/2;
 IF Delta_d<=3*Dext THEN K1:=Power(Delta_d/(3*Dext),0.6-(Delta_d-3*Dext)/(6*Dext))
 ELSE K1:=Power(Delta_d/(3*Dext),0.34);
 f:=3.36*K1*D1*SQR(Ua/U0);

 {display of f}
 //For the previous set of hypothesis, the result is f=966 mm;
 WRITELN('f=',f:4:0,' mm');
END;
```